Chapter 6

Type-2 Fuzzy Logic Applications in Solar Energy Technologies: A Comprehensive Overview

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Abstract

This book chapter gives an overview of Type 2 Fuzzy Logic (T2FL) and its application in solar energy research. Through a comprehensive investigation, both the applications and limitations of T2FL in the context of solar energy systems are illuminated. It examines the role of T2FL methods in managing uncertainty and draws insights from contemporary academic literature. This book chapter contributes to a differentiated understanding of T2FL methods and their impact in the field of solar technologies. The chapter examines how T2FL helps improve the efficiency of solar energy forecasting models, overcome power control challenges, and improves solar system performance. Furthermore, the integration of T2FL with MCDM (Multi-Criteria Decision-Making) methods for evaluating renewable energy alternatives is discussed. By providing practical insights and addressing potential challenges, this chapter seeks to advance knowledge in these interrelated areas. The content presented here aims to provide valuable perspectives for both academic and professional audiences and to position the T2FL principles in the dynamic landscape of solar energy. In summary, the holistic view presented here enriches the discourse on T2FL methods and highlights their importance in the specific area of solar energy research.

1. Introduction

Traditional methodologies, designed around precise boundaries, often struggle to grapple with the multifaceted challenges of data generated by humans, inherently laden with imprecision and vagueness. These classical approaches fall short when confronted with the intricate nature of human cognitive processes. The term 'approximate' is commonly used in everyday discourse to pragmatically represent numerical values that are uncertain

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or undefined, especially for quantities not based on precise measurements or well-defined parameters. However, the expressed, understood, and actual values may vary due to differences in both the formulation of the magnitude indicated by this 'approximate' expression by the speaker and the interpretation by the recipient. In situations characterized by inherent uncertainties, such as measurement inaccuracies or linguistic ambiguities, the indicated magnitude inherently incorporates an element of unpredictability.

Fuzzy Logic (FL) is a powerful tool that effectively addresses the complexities and uncertainties found in the real world. FL possesses the unique capability to navigate the subtleties and complexities inherent in various real-world scenarios. FL excels at faithfully replicating human reasoning, embracing the inherent ambiguities of human thought processes. This robust methodology mirrors the intricate nature of human reasoning and thrives in scenarios characterized by imprecision and vagueness, making it a versatile tool with diverse applications. FL has emerged as a prominent paradigm in the realm of scientific research, finding applications across a diverse array of academic disciplines. It has positioned itself as a sophisticated algorithm with relevance to intelligent systems, garnering extensive attention and recognition in academic investigations. This is notably apparent in its utilization across various engineering domains [1], [2], medical sectors [3]–[5], and decision support systems, underscoring its versatility and efficacy as a paradigm. FL, closely related to fuzzy set theory, transcends the understanding of complex systems. The evolution and implementation of fuzzy set theory have significantly broadened our capacity to model intricate real-world phenomena, offering invaluable insights into the realms of decision-making and artificial intelligence. It plays a crucial role in shaping the advancement of artificial intelligence by empowering machines to address tasks demanding problem-solving capabilities akin to those of humans, navigating the challenges posed by uncertainty. FSs excel in handling ambiguity in calculations and the modeling and control of complex systems, proving to be invaluable in both academic research and practical applications.

In order to improve the interpretive clarity and usefulness of naturally ambiguous concepts and to represent their heuristic meaning in a way that takes into account a variety of viewpoints, this expressed magnitude can be represented as a fuzzy set (FS). This approach leads to the creation of fuzzy numbers (FNs) in accordance with the mathematics of fuzzy logic, building upon the definition of FSs [6], [7]. To address the limitations of conventional set theory, Zadeh introduced fuzzy set theory in 1965. Further expounded upon by Zadeh in 1996, this theoretical framework has significantly revolutionized our ability to address uncertainty and vagueness across diverse fields of study.

As we navigate the escalating complexities of uncertainty, academic focus organically shifts towards Type-2 Fuzzy Sets (T2FS), drawing inspiration from the seminal contributions of Mendel and Wu in 2002 [8]. The influential work of Mendel and Wu identified four main sources of uncertainty within FL: the use of technical terminology in rule antecedents and consequences, introducing inherent uncertainty; consequents depicting a spectrum of potential outcomes, often illustrated as histograms; measurements susceptible to the influence of noise; and the presence of noisy characteristics in tuning data, further complicating the landscape.

Type 1 fuzzy sets (T1FSs), a widely used subtype of FL, successfully navigate areas of uncertainty and imprecision compared to traditional approaches. T1MFs constructed according to the T1FS structure are favored for their practicality, offering a straightforward conceptual grasp and computational efficiency, making them the preferred choice for converting input variables into their fuzzy counterparts. Among the MF forms that are available, Gaussian, trapezoidal and triangular configurations are prevalent in practical applications. In the realm of scientific inquiry, T1FSs operate under the assumption of a definitive certainty regarding the MF shape designed by the researcher to suit a specific problem. Nevertheless, a notable challenge arises as T1MFs fall short in accommodating the uncertainty emerging from their inability to comprehensively articulate the MFs shape in practical contexts. This challenge prompts a deeper consideration for employing advanced methodologies, such as T2FS, renowned for its capability to address the inherent uncertainty in MF shapes. As we navigate the growing intricacies of uncertainty, scholarly attention naturally shifts towards T2FSs, inspired by the seminal contributions of Mendel and Wu in 2002 [8]. T2FSs excel in modeling uncertainty within the MF by representing its boundaries with fuzziness. The fuzzy representation of the boundaries of Type 2 membership functions (T2MFs) provides the opportunity to model uncertainties. Essentially, T2MFs are therefore inherently blurry. Furthermore, adding a third dimension to T2MFs increases the ability to model uncertainty levels. T2FSs emerge as the preferred choice for managing multifaceted uncertainties, as emphasized by Mendel and Wu in 2002 [8]. However, comprehending and applying T2FSs poses challenges, as we will explore. Unfortunately, T2FS are more complicated to use and understand compared to their Type 1 counterparts, which may contribute to their limited distribution.

The upcoming chapters aim to illuminate the complexities of FL, meticulously examining the strengths and limitations inherent in both methodologies, with a specific focus on their relevance to solar energy. Our goal is that, upon completion of this chapter, professionals and scholars immersed in solar energy research will possess the essential knowledge to make insightful decisions regarding the practical application of FL in the context of solar technologies. The enduring scholarly and industrial fascination with FSs continues to drive extensive research efforts across diverse domains, particularly within the ever-evolving landscape of solar energy.

This chapter's main goal is to offer insights regarding Type-2 Fuzzy Logic's (T2FLs) contemporary applications in the field of solar energy, with a specific focus on Type-2 Fuzzy Numbers (T2FNs) and Type-2 Fuzzy Logic Systems (T2FLSs). Fuzzy logic finds two significant applications in fuzzy numbers and fuzzy systems. The initial stage of the book chapter imparts knowledge about T2FL theory. At this juncture, information about T2FNs and their arithmetic operations will be presented through an examination of General Interval Type-2 Triangular Fuzzy Numbers (GIT2TFNs), a type of T2FN, for enhanced comprehensibility. Subsequently, to facilitate understanding, T2FLSs will be explained comparatively with Type-1 Fuzzy Logic Systems (T1FLSs).

This book chapter is divided into multiple sections, each focusing on a different facet of T2FNs and T2FLSs in their modern solar energy applications. In the introductory section, the focus is on the theory of T2FL, highlighting its capabilities in dealing with uncertainty scenarios and emphasizing the distinctions from T1FL. The subsequent chapter, titled "Brief Introduction of the Type-2 Fuzzy Set Calculations," thoroughly examines the fundamental concepts of T2FL, making sure that its mathematical concepts and principles are fully understood.. This section consists of two sub-sections, namely "Interval Type-2 Fuzzy Logic Systems" and "Interval Type-2 Fuzzy Numbers." In these subsections, concepts such as Interval Type-2 Fuzzy Numbers (IT2FNs) and Interval Type-2 Fuzzy Logic Systems (IT2FLS) are explored in-depth, providing a clear explanation of basic concepts and processes and laying the groundwork for the topics to be addressed in the subsequent chapters. The following sections, "Applications of T2FLSs in solar energy" and "Applications of T2FNs in solar energy," sequentially delve into the roles of T2FN and T2FLS in the complexities and uncertainties in solar energy research, highlighting novel applications in the literature. These chapters offer a comprehensive understanding of how T2FL is applied in the field of solar energy, providing readers with insights into the latest developments. Additionally, they focus on a broad range of application areas, extending from decision-making processes involving IT2FNs to the application of T2FLSs in solar energy prediction models.

2. Brief Introduction of the Type-2 Fuzzy Set Calculations

The Boolean system uses a value system where 1 represents absolute truth and 0 represents absolute falsehood, as exemplified by the MF expressions below. These expressions illustrate the binary nature of the Boolean system, capturing the essence of truth and falsehood within the defined context.

$$\mu_A(x) = 1 \ if \ x \in A \tag{1}$$

$$\mu_A(x) = 0 \ if \ x \notin A \tag{2}$$

Fuzzy set theory allows the representation of truthfulness or falsity degrees through membership degrees. This metric determines the degree to which element 'x' is a member of a fuzzy set, represented by the letter 'A'. There is a wide range of truthfulness values in FL, from complete falsity to complete truthfulness. Membership values represent partial degrees of truthfulness and falsity and fall between 0 and 1. These degrees, which simplify as a judgment call on an element's degree of association with the set, are computed using a MF. A membership value of 0 indicates total non-membership and absolute falsity. On the other hand, a membership value of 1 denotes complete honesty and represents all members of the set. This nuanced approach to truth values, facilitated by fuzzy set theory, is instrumental for addressing uncertainty and vagueness in diverse applications, providing a sophisticated framework for decision-making and problem-solving.

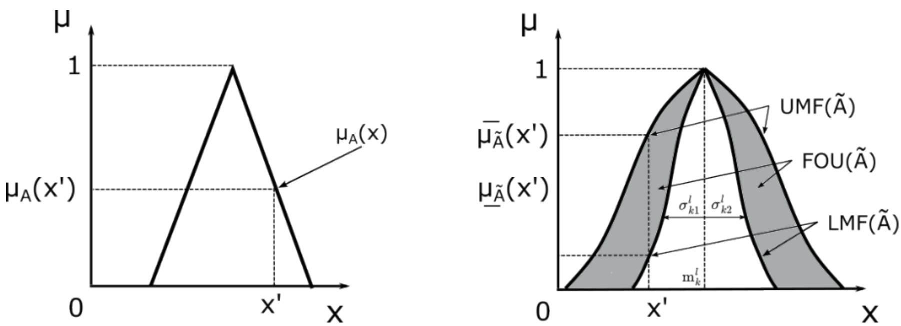
As a result, Lotfi A. Zadeh introduced the concept of type-n fuzzy sets, encompassing T1FSs and T2FSs [9]. These sets form the basis of fuzzy theory, enabling the computation of membership degrees. Particularly, when dealing with a T1FS denoted as 'A' equation (3) is employed to ascertain membership characteristics. The membership function is denoted as $\mu_A(x)$, where $x' \in X$, expressed as equation (4).

$$A = \{(x, \mu_A(x) \mid x \in X \ \} \tag{3}$$

$$\mu_A(x) \rightarrow [0,1] \tag{4}$$

In simple terms, converting crisp input variables into fuzzy variables is the method's basic operation. In order to express the degree of membership of the variable "x" to the FS, the MF acts as a defining factor by allocating a real numerical value in the range [0-1]. Numerous MFs have been developed to accommodate different types of FSs, with T1MFs occupying a predominant position in terms of recognition and general use. In contrast, T2FS are three-dimensional structures and are based on principle of Zadeh's extension for fundamental operations like intersection, complement and union. Thus, in comparison to T1FS, comprehending T2FS typically involves more intricate and lengthy computations. Their wider adoption in a variety of applications has been hampered by their increased complexity.

We use linear functions to determine the boundaries of T1MFs, as shown in Figure 1-a, while the boundaries of T2MFs are determined as the interval between the upper and lower membership functions (UMF and LMF), as shown in Figure 1-b. The upper membership function (UMF) is represented by an overline, while the lower membership function (LMF) is indicated by an underline. The "Footprint of Uncertainty" (FOU), as depicted in Figure 1-b, is the area that exists between these fuzzy borders. The UMF and the LMF are both essentially two T1MFs that define the outer limits of the FOU within a T2MF. The highest degree of membership in the FOU subset is characterized by the UMF, whereas the lowest degree of membership in the subset is characterized by the LMF.



(a) *(b)*

Figure 1: Illustrations of Membership Functions for Type-1 Triangular (a) and Type-2 Gaussian (b) [10]

Mendel (2014) identified four mathematical representations for a T2FS: a) A group of individual points, as shown by Equation (5). b) As shown by Equation (6), a union of vertical slices over the whole domain X, where each vertical slice is a T1FS (a secondary MF). c) A union of wavy slices, each of which stands for a T2FS embedded in it. d) A fuzzy union of horizontal slices over [0, 1], where each horizontal slice resembles an IT2FS elevated to α [11]. In this book chapter, the symbol Ã was used to denote T2FS, which can be formulated as follows.

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall\, x \in X, \forall u \in J_x \subseteq [0,1]\}; \quad 0 \leq \mu_{\tilde{A}}(x,u) \leq 1 \tag{5}$$

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)}, \quad J_x \subseteq [0,1] \tag{6}$$

In this particular context, the primary variable of a T2FS is denoted by x, while the secondary variable is represented by u. Additionally, J designates the secondary MF. For a specific value of of $x = x'$, $\mu_{\tilde{A}}(x',u)$ signifies the vertical cross-section of $\mu_{\tilde{A}}(x,u)$, and the formal expression of the T2MF of $\tilde{A}$ is presented in equation (7). Here, $x \in X$, $u \in J_{x'} \subseteq [0,1]$, and $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$, as described by Mendel and John [12]. Additionally, $f_{x'}(u)$ signifies the magnitude of the secondary MF, where $f_{x'}(u) \subseteq [0,1]$.

$$\mu_{\tilde{A}}(x = x',u) \equiv \mu_{\tilde{A}}(x') = \int_{u\in J_{x'}} \frac{f_{x'}(u)}{u}, \quad J_{x'} \subseteq [0,1] \tag{7}$$

When $\mu_{\tilde{A}}(x,u) = 1$,

Ã is recognized as an IT2MF. The third dimension in general T2FSs is often considered redundant, providing no additional information, and is thus treated as a special case. Specifically, an IT2FS is viewed as a distinct subtype within the broader category of T2FSs. Due to its user-friendly nature and minimal computational demands, IT2FSs find frequent application. In these sentences the third dimension is considered uninformative and is taken to stay unchanged. As a result, only the FOU is utilized to characterize IT2FSs, and the third dimension is dismissed. The MF, influenced by the distinctive traits of T2FS boundaries, can adopt values within the [0-1] range along a vertical continuum between the upper-bound $\overline{\mu_{\tilde{A}}}(x')$ and the lower-bound $\underline{\mu_{\tilde{A}}}(x')$, deviating from the crisp numbers utilized in the context of T1FSs. T2FS and IT2MF are expressed in equation 8 and equation 9, respectively.

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \frac{1}{(x,u)}, \quad J_x \subseteq [0,1] \tag{8}$$

$$\mu_{\tilde{A}}(x = x',u) \equiv \mu_{\tilde{A}}(x') = \int_{u\in J_{x'}} \frac{1}{u}, \quad J_{x'} \subseteq [0,1] \tag{9}$$

The values representing the highest membership degree of the FOU for the UMF and the portion with the lowest membership degree for the LMF can be expressed by equations (10) and (11), respectively. Utilizing equation (12) for an arbitrary T2MF interval $\tilde{Q}$, membership degrees for its lower and upper bounds can be calculated.

$$\overline{\mu}_{\tilde{A}}(x) \equiv \overline{FOU}(\tilde{A}), \quad \forall\, x \in X \tag{10}$$

$$\underline{\mu}_{\tilde{A}}(x) \equiv \underline{FOU}(\tilde{A}), \quad \forall\, x \in X \tag{11}$$

$$\mu_{\tilde{Q}_k^l}(x_k) = \int_{q^l \in \left[\underline{\mu}_{\tilde{Q}_k^l}(x_k), \bar{\mu}_{\tilde{Q}_k^l}(x_k)\right]} 1/q^l \tag{12}$$

The MFs in Interval Type-2 Fuzzy Sets (IT2FSs) can be of various shapes, such as triangular, trapezoidal, Gaussian and sigmoid types similar to those used in T1FSs. If we symbolize the Gaussian Membership Function as $\tilde{g}_k^l(x_k)$, an analogous equation can be articulated, following the framework presented by Liang and Mendel [13]. In the case of a Gaussian MF with a fixed mean m_k^l and a variable standard deviation falling within the interval [σ_{k1}^l, σ_{k2}^l], the MF is denoted by Equation (14).

$$\mu_{\tilde{Q}_k^l}(x_k) = \int_{v^l \in \left[\underline{\mu}_{\tilde{X}_k^l}(x_k), \bar{\mu}_{\tilde{X}_k^l}(x_k)\right]} 1/v^l \tag{13}$$

$$\mu_k^l(x_k) = exp\left[-\frac{1}{2}\left(\frac{x_k - m_k^l}{\sigma_k^l}\right)^2\right], \quad \sigma_k^l \in \left[\sigma_{k1}^l, \sigma_{k2}^l\right] \tag{14}$$

2.1. Interval Type-2 Fuzzy Logic Systems

The typical configuration of a T2FLS is illustrated in Figure 2. The structure of a T1FLS encompasses fuzzification, a fuzzy rule base, a fuzzy inference engine and defuzzification processes. Comparatively, the structure of a T2FLS closely resembles that of T1FLSs, with the notable distinction that the type reducer, facilitating the conversion from T2FSs to T1FSs, precedes the defuzzifier. Additionally, the architecture of a T2FLS aligns with the type-1 form, differing primarily in having at least one fuzzy set in the rule base designated as type-2. Conversely, the output of a T2FLS inference engine consists of T2FS, necessitating the use of a type reducer to transform them into T2FSs before feeding them into the defuzzification process [14].

The overall framework of a T2FLS is presented in Figure 2. The structure of a T1FLS includes key components such as fuzzification, a fuzzy rule base, a fuzzy inference engine, and defuzzification. Notably, the architecture of an IT2FLS closely resembles that of T1FLSs, with a distinctive variation: In T2FLS using T2FSs instead of T1FSs, the presence of a type reducer positioned before the defuzzifier to reduce T2FSs to T1FSs facilitates the transformation from T2FSs to T1FSs. Furthermore, the T2FLS structure aligns with the type-1 form, differing primarily in the inclusion of at least one fuzzy set in the rule base designated as type-2. Conversely, the output of a T2FLS inference engine comprises T2FSs, necessitating a type reducer to convert them into T1FSs before feeding them into the defuzzification process [15].

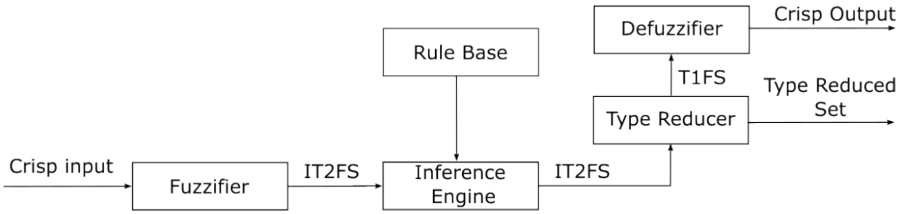


Figure 2: General architecture of the IT2FLS [10]

The operational principle of an IT2FLS can be elucidated as follows: the crisp inputs undergo mapping by the fuzzifier, resulting in input IT2FL sets. The fuzzifier of an IT2FLS may be of singleton or non-singleton type. In the case of a singleton fuzzifier, denoted as (i=1, ..., p), the input fuzzy set $\tilde{A}_{x_i}$ possesses only one point of nonzero membership, expressed as follows

$$\tilde{A}_{x_i}(x_i) = \begin{cases} 1, & x_i = x_i' \\ 0, & x_i \neq x_i' \end{cases} \tag{15}$$

The rule base of T2FLS is akin to that of T1FL, but in T2FLS, FSs are shaped by expert knowledge and articulated through their antecedents and consequents. The antecedents and/or the consequents are expressed by IT2FSs, yet the rules maintain a similarity to T1FLS. Mamdani rules, which result in IT2FSs, and Takagi and Sugeno (TSK) rules, which result in net functions of inputs, are the two different types of rules for IT2FLSs. [16]. IT2FSs do not impact the rule base; the rule structure of IT2FLSs aligns with that of T1FLSs. The Mamdani rule base is generally widely used because it integrates expert knowledge easily, simply, and adaptably. The form for the l th rule of an IT2FLS that has M rules, p inputs $x_1 \in X_1, \ldots, x_p \in X_p$, and one output $y \in Y$ is presented as shown in Equation (16).

$$R^n = IF\ x_1\ is\ \tilde{F}_1^l\ and \ldots and\ x_p\ is\ \tilde{F}_p^l\ THEN\ y\ is\ \tilde{G}^l \tag{16}$$

The fuzzy inference engine, a crucial component in FL reasoning, is employed to generate fuzzy outputs based on the fuzzified inputs. This phase involves the inference engine determining, for each IF-THEN rule, the outputs that correspond to the fuzzified inputs that were computed in the previous fuzzifier stage. The fired rules are consolidated by the inference engine, followed by a mapping process from input T2FSs to output T2FSs. It produces the IT2FS consequents from the IT2FS antecedents by combining all the fired rules. The meet and join operations are employed to connect multiple antecedents in each rule and integrate all the rules. As can be seen below, the result of the *k* th input and the matching antecedent operations in the *l* th rule is an interval [17].

$$\mu_{F_k^l}(x_k) = \left[\underline{\mu}_{\tilde{F}_k^l}(x_k), \overline{\mu}_{\tilde{F}_k^l}(x_k)\right] \tag{17}$$

To employ a T2FLS in practical applications, obtaining the precise output in crisp form becomes imperative. To achieve this, it is necessary to obtain the type-reduced set, which is an interval that represents a T2FS's center of gravity. The type reducer processes the T2FS outputs from the inference engine, executing a calculation that results in T1FSs known as the type-reduced sets. Subsequently, the type-reduced sets undergo processing by the defuzzifier to yield crisp outputs that must be directed to the actuators. The Karnik-Mendel (KM) algorithm, which is based on iterative operations and functions as an extension of a type-1 defuzzification procedure, is the most commonly used approach for this purpose [18].

To transform the interval set that is acquired after the type reduction operation into a precise number, defuzzification is then required. Equation (18) illustrates how to perform this process, which involves merely calculating the average of the range's left and right endpoints [12].

$$y_{out} = \frac{y_r + y_l}{2} \tag{18}$$

2.2. Interval Type-2 Fuzzy Numbers

Interval Type-2 Fuzzy Numbers (IT2FNs), with their two-dimensional membership function structure, enable a more comprehensive handling of uncertainties and complexities in the model, significantly reducing complexity in problem-solving. Evaluated as two-dimensional due to the absence of information in its third dimension, IT2FNs have facilitated and simplified mathematical operations. The computational efficiency of IT2FNs has positioned them at the forefront of scientific inquiry, particularly when contrasted with the computational intricacies associated with T2FNs.

In the context of IT2FSs, denoted as $\tilde{A}$ and bounded by LMF and UMF represented as $\tilde{A} = (A^l, A^u)$, specific categorizations have been established. An IT2FS achieves the classification of a Perfect Interval Type-2 Fuzzy Number (PIT2FN) if both its UMF and LMF are Type-1 Fuzzy Numbers (T1FNs). Alternatively, if the UMF is a T1FN while the LMF is a Fuzzy Sub Number (FSN), the IT2FS is referred to as IT2FN. When both UMF and LMF are FSNs, the IT2FS is called an Interval Type-2 Fuzzy Sub Number (IT2FSN).

General Interval Type-2 Trapezoidal Fuzzy Numbers (GIT2TrFNs) play a fundamental role in arithmetic operations and the ranking of IT2FNs, which are characterized by rectilinear boundaries, as shown in Figure 3. In this framework, GIT2TFNs appear as a separate case within the GIT2TrFN category.

For arithmetic operations and ranking among IT2FNs with linear boundary profiles, GIT2TrFNs can be used as an exemplary model as shown in Figure 3.. It is noteworthy that from the information provided for GIT2TrFN, analogous procedures for GIT2TFNs can be easily derived.

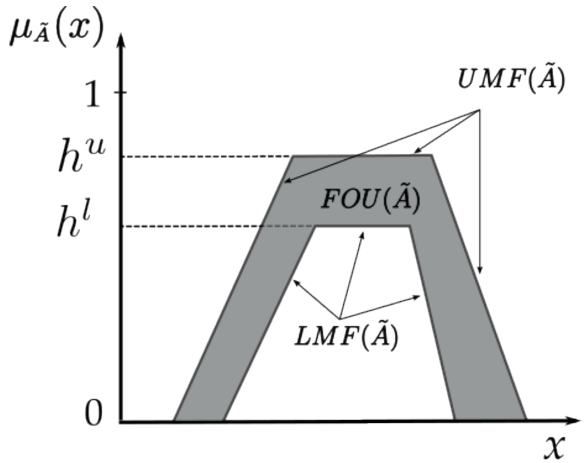


Figure 3: UMF, LMF and FOU representation for a GIT2TrFN with heights of h^l and h^u [19]

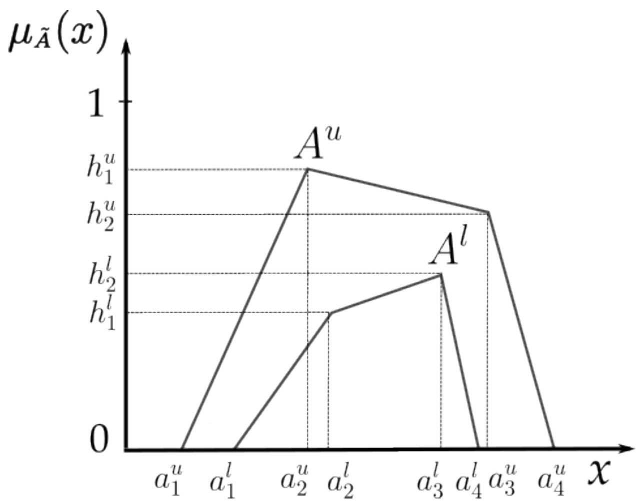


Figure 4: A^u and A^l functions of a GIT2TrFN and representation of their heights [19]

The GIT2TrFN depicted in Figure 4 can be mathematically expressed as in $\tilde{A} = (A^l, A^u) = \left((a_1^l, a_2^l, a_3^l, a_4^l, h_1^l, h_2^l), (a_1^u, a_2^u, a_3^u, a_4^u, h_1^u, h_2^u)\right)$ and can be characterized for its MFs, denoted as A^l and A^u, as follows. When $h_1^l = h_2^l = h^l$, $h_1^u = h_2^u = h^u$, and $h_1^l = h_2^l = h_1^u = h_2^u = 1$ conditions are present, these numbers are respectively referred to as Interval Type-2 Flat Trapezoidal Fuzzy Number (IT2FTrFN) and Perfect Interval Type-2 Trapezoidal Fuzzy Number (PIT2TrFN).

$$\mu_{A^l}(x) = \begin{cases} \mu_{A_1^l}(x) = h_1^l \frac{x - a_1^l}{a_2^l - a_1^l} & a_1^l \le x \le a_2^l \\ \mu_{A_2^l}(x) = (h_2^l - h_1^l)\frac{x - a_2^l}{a_3^l - a_2^l} + h_1^l & a_2^l \le x \le a_3^l \\ \mu_{A_3^l}(x) = h_2^l \frac{a_4^l - x}{a_4^l - a_3^l} & a_3^l \le x \le a_4^l \\ \mu_{A_4^l}(x) = 0 & x \le a_1^l, x \ge a_4^l \end{cases} \quad (19)$$

and

$$\mu_{A^u}(x) = \begin{cases} \mu_{A_1^u}(x) = h_1^u \frac{x - a_1^u}{a_2^u - a_1^u} & a_1^u \le x \le a_2^u \\ \mu_{A_2^u}(x) = (h_2^u - h_1^u)\frac{x - a_2^u}{a_3^u - a_2^u} + h_1^u & a_2^u \le x \le a_3^u \\ \mu_{A_3^u}(x) = h_2^u \frac{a_4^u - x}{a_4^u - a_3^u} & a_3^u \le x \le a_4^u \\ \mu_{A_4^u}(x) = 0 & x \le a_1^u, x \ge a_4^u \end{cases} \quad (20)$$

For the case where

$$\tilde{A} = (A^l, A^u) = \left((a_1^l, a_2^l, a_3^l, a_4^l, h_{1A}^l, h_{2A}^l), (a_1^u, a_2^u, a_3^u, a_4^u, h_{1A}^u, h_{2A}^u)\right)$$

$$\tilde{B} = (B^l, B^u) = \left((b_1^l, b_2^l, b_3^l, b_4^l, h_{1B}^l, h_{2B}^l), (b_1^u, b_2^u, b_3^u, b_4^u, h_{1B}^u, h_{2B}^u)\right)$$

and k is crisp, certain arithmetic and ranking operations for GIT2TrFNs are defined as follows [20]–[22]:

Addition Operation:

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (A^l \oplus B^l, A^u \oplus B^u) \\ &= \left(a_1^l + b_1^l, a_2^l + b_2^l, a_3^l + b_3^l, a_4^l + b_4^l, \min\{h_{1A}^l, h_{1B}^l\}, \min\{h_{2A}^l, h_{2B}^l\}\right), \\ &\quad (a_1^u + b_1^u, a_2^u + b_2^u, a_3^u + b_3^u, a_4^u + b_4^u, \min\{h_{1A}^u, h_{1B}^u\}, \min\{h_{2A}^u, h_{2B}^u\}) \end{aligned} \quad (21)$$

Subtraction Operation:

$$\begin{aligned} \tilde{A} \ominus \tilde{B} &= (A^l \ominus B^l, A^u \ominus B^u) \\ &= \left(a_1^l - b_4^l, a_2^l - b_3^l, a_3^l - b_2^l, a_4^l - b_1^l, \min\{h_{1A}^l, h_{1B}^l\}, \min\{h_{2A}^l, h_{2B}^l\}\right), \\ &\quad (a_1^u - b_4^u, a_2^u - b_3^u, a_3^u - b_2^u, a_4^u - b_1^u, \min\{h_{1A}^u, h_{1B}^u\}, \min\{h_{2A}^u, h_{2B}^u\}) \end{aligned} \quad (22)$$

Multiplication Operation:

$$\tilde{A} \otimes \tilde{B} = \left(\left(c_1^l, c_2^l, c_3^l, c_4^l, h_{1C}^l, h_{2C}^l\right), \left(c_1^u, c_2^u, c_3^u, c_4^u, h_{1C}^u, h_{2C}^u\right)\right) \quad (23)$$

where

$$\begin{aligned}
c_1^l &= min\{a_1^l \times b_1^l, a_1^l \times b_4^l, a_4^l \times b_1^l, a_4^l \times b_4^l\} \\
c_2^l &= min\{a_2^l \times b_2^l, a_2^l \times b_3^l, a_3^l \times b_2^l, a_3^l \times b_3^l\} \\
c_3^l &= max\{a_2^l \times b_2^l, a_2^l \times b_3^l, a_3^l \times b_2^l, a_3^l \times b_3^l\} \\
c_4^l &= max\{a_1^l \times b_1^l, a_1^l \times b_4^l, a_4^l \times b_1^l, a_4^l \times b_4^l\} \\
h_{1C}^l &= min\{h_{1A}^l, h_{1B}^l\} \\
h_{2C}^l &= min\{h_{2A}^l, h_{2B}^l\} \\
c_1^u &= min\{a_1^u \times b_1^u, a_1^u \times b_4^u, a_4^u \times b_1^u, a_4^u \times b_4^u\} \\
c_2^u &= min\{a_2^u \times b_2^u, a_2^u \times b_3^u, a_3^u \times b_2^u, a_3^u \times b_3^u\} \\
c_3^u &= max\{a_2^u \times b_2^u, a_2^u \times b_3^u, a_3^u \times b_2^u, a_3^u \times b_3^u\} \\
c_4^u &= max\{a_1^u \times b_1^u, a_1^u \times b_4^u, a_4^u \times b_1^u, a_4^u \times b_4^u\} \\
h_{1C}^u &= min\{h_{1A}^u, h_{1B}^u\} \\
h_{2C}^u &= min\{h_{2A}^u, h_{2B}^u\}
\end{aligned} \quad (24)$$

Scalar Multiplication Operation:

$$if\ k \geq 0 \quad k\tilde{A} = (kA^l, kA^u) = \left((ka_1^l, ka_2^l, ka_3^l, ka_4^l, kh_{1A}^l, kh_{2A}^l), (ka_1^u, ka_2^u, ka_3^u, ka_4^u, kh_{1A}^u, kh_{2A}^u)\right) \quad (25)$$

$$if\ k \leq 0 \quad k\tilde{A} = (kA^l, kA^u) = \left((ka_4^l, ka_3^l, ka_2^l, ka_1^l, kh_{1A}^l, kh_{2A}^l), (ka_4^u, ka_3^u, ka_2^u, ka_1^u, kh_{1A}^u, kh_{2A}^u)\right) \quad (26)$$

Ranking of IT2FNs: The IT2FS ranking is indispensable due to its widespread use in decision-making processes and its critical role in facilitating a deeper study of fuzzy systems. The importance of the IT2FN ranking cannot be overlooked as it plays an important role in decision-making processes and allows for a further investigation of fuzzy systems. Although the field of ranking methods on IT2FS is noticeably narrow and requires new approaches, similarity and uncertainty measures have drawn broad methodology. While there exists extensive methodology and research interest in similarity and uncertainty measures for IT2FSs, the field of ranking methods is constrained, requiring fresh perspectives and innovative solutions [20], [22]–[25].

The ranking method proposed by Lee and Chen for IT2TrFSs for the interval type-2 trapezoidal fuzzy set expressed as $\tilde{A}_i = (A_i^l, A_i^u) = \left(\left(a_{i1}^l, a_{i2}^l, a_{i3}^l, a_{i4}^l, h_{1A_i}^l, h_{2A_i}^l\right), \left(a_{i1}^u, a_{i2}^u, a_{i3}^u, a_{i4}^u, h_{1A_i}^u, h_{2A_i}^u\right)\right)$

is presented below [22]. In the equations, the ranking value is denoted as Rank($\tilde{A}$).

$$Rank(\tilde{A}) = M_1(\tilde{A}_i^l) + M_1(\tilde{A}_i^u) + M_2(\tilde{A}_i^l) + M_2(\tilde{A}_i^u) + M_3(\tilde{A}_i^l) + M_3(\tilde{A}_i^u) - \frac{1}{4}\left(S_1(\tilde{A}_i^l) + S_1(\tilde{A}_i^u) + S_2(\tilde{A}_i^l) + S_2(\tilde{A}_i^u) + S_3(\tilde{A}_i^l) + S_3(\tilde{A}_i^u) + S_4(\tilde{A}_i^l) + S_4(\tilde{A}_i^u)\right) + H_1(\tilde{A}_i^l) + H_1(\tilde{A}_i^u) + H_2(\tilde{A}_i^l) + H_2(\tilde{A}_i^u) \quad (27)$$

$$M_p(\tilde{A}_i^j) = \frac{a_{ip}^j + a_{ip+1}^j}{2}$$

denotes the average of the elements a_{ip}^j and $a_{i(p+1)}^j$ for $1 \leq p \leq 3$,

$$S_q(\tilde{A}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1}\left(a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j\right)^2}$$ denotes the standart deviation for $1 \leq q \leq 3$,

$H_p(\tilde{A}_i^j)$ denotes the membership value of the element $a_{i(p+1)}^j$ in the trapezoidal MF $\tilde{A}_i^j$ for $1 \leq p \leq 2$, $j \in \{U, L\}$ and $1 \leq i \leq n$

3. Applications of T2FLSs in solar energy

In recent years, the utilization of T2FLSs has gained prominence in the realm of solar energy research, offering a robust framework for addressing uncertainties and complexities inherent in various aspects of solar energy studies. This text explores the diverse scientific domains where T2FLSs have found application within solar energy investigations, providing a comprehensive overview of their contributions.

The integration of solar energy with other Renewable Energy Sources (RESs), such as wind or biomass, has been facilitated through the application of type-2 fuzzy control systems (T2FLCs). This integration aims to create more reliable and efficient hybrid energy solutions. The use of electric vehicles is rapidly expanding in contemporary times, with users frequently charging their electric vehicles at home. Beheshtikhoo et al. have designed a T2FLC for the energy management system of a smart home equipped with an electric vehicle charging station and a RES. Their proposed algorithm effectively controls a small hybrid renewable energy system, combining photovoltaic (PV), vertical axis wind turbines, fuel cells, electric vehicles and energy storage systems, in a climate-independent manner. In their study, they observed that the controller could significantly reduce daily electricity consumption from the grid, electricity costs and the peak-to-average ratio of the system [26].

To address the challenges posed by continuous and severe output power variations in the integration of solar energy power systems with each other or with traditional and other renewable power sources, controllers are employed to manage load frequency control problems. Soliman et al. have proposed a T2FLC, not only to mitigate the impact of solar irradiance changes on the power system but also to regulate the output of the solar park on cloudy days, instead of relying on maximum power point trackers. In order to improve the suggested controller's dynamic performance, a meta-heuristic, nature-inspired optimization algorithm such as the Whale Optimization Algorithm (WOA) has been suggested for offline tuning of controller gains [27]. To reduce the total harmonic content in grid-tied solar energy systems using boost converters for integrated solar multi-level inverters, Gopinath et al. have proposed a solar-powered cascaded topology with Interval Type-2 Fuzzy Logic Controller (IT2FLC). This topology aims to predict the switching frequency for multi-level inverters, enhancing performance in minimizing total harmonic distortion and ensuring a stable output power [28]. In the solar PV integrated power system, another challenge related to power control is the reduction of Low-Frequency Oscillations (LFOs), which impact the stability of the power system. Paital et al. have proposed a robust Power System Stabilizer (PSS) based on Interval Type-2 Fuzzy Proportional Integral Derivative (IT2FPID), considering uncertainties, aiming to address and optimize the solution to this problem in their study [29].

T2FLS have proven valuable in improving solar energy prediction models. By accommodating uncertainties related to factors like sunlight intensity and weather conditions, these systems enhance the accuracy of energy production forecasts. The random nature of RESs often leads to an inability to provide continuous energy, prompting their frequent combination with energy storage systems or other energy sources. To overcome this weakness in RESs, predictions can be made using meteorological data, specifically reducing uncertainties in the sizing of energy projects. Jafarzadeh et al. utilized fuzzy logic to model the predicted temperature and irradiance data, employing type-1 and Interval Type-2 Takagi-Sugeno-Kang (TSK) fuzzy systems for modeling and forecasting solar power plants. The significant advantage of using IT2FL models is that, in addition to predicting the power generation value, it provides an uncertainty range. Information related to uncertainty in the prediction enables operators to develop effective bidding strategies in the electricity market [30].

Researchers utilize type-2 fuzzy systems to address optimization challenges in the design and operation of solar energy systems. These systems provide solutions for complex problems involving multiple objectives and constraints. Hydrogen energy, a green energy type, becomes even more crucial for sustainable development when produced by using a RES as solar energy. Benghanem et al. investigated the power loss and efficiency of a PV-electrolyzer system comprising a PV source and electrolyzer stack in both direct and indirect-coupled scenarios. To enhance energy transfer, they proposed a type-2 fuzzy logic controller to regulate the operating point of the PV array by adjusting the duty cycle of the control signal of a buck converter placed between the PV array and the electrolyzer stack. In the study, the indirectly-coupled hydrogen system achieved a higher energy transfer rate by ensuring the continuous extraction of the maximum power from the PV source [31].

Researchers have integrated T2FLS into studies aimed at enhancing the efficiency of solar energy systems. By modeling the interactions of various parameters and variables, these systems assist in optimizing the performance of solar installations. In the field of energy production using PVs, various Maximum Power Point (MPP) tracking methods have been developed to achieve maximum energy output. Key environmental conditions such as solar radiation and temperature are critical to achieving maximum energy and these conditions are often uncertain and irregular. The traditional method cannot track the Maximum Power Point (MPP) when solar irradiance and/or panel temperature changes rapidly because rapid changes in PV current and PV voltage cannot be distinguished under real environmental conditions. In Kayisli's study, a super twist slide mode controller was developed for maximum power point tracking. It has been adapted to a Type 2 fuzzy cluster system to mitigate chatter problems, and the parameters of both the super twisting sliding mode and the Type 2 fuzzy cluster have been optimized for improved performance [32]. The traditional incremental conductivity (AIC) method cannot efficiently track the maximum power point (MPP) because it is unable to distinguish rapid changes in photovoltaic (PV) current and voltage under actual environmental conditions. Gani et al. found that their approach, based on the combination of AIC and T2FL, provides a significant advantage in terms of maximum power transfer in long-term weather conditions with frequent cloudburst occurrence over a 40-month test period [33]. Pandey et al. have designed an Asymmetric Interval Type-2 Fuzzy Logic controller-based MPPT to cope with sudden irradiance changes commonly encountered in PV applications, which adversely affect the operation of the Maximum Power Point Tracking (MPPT) system. They compared the results with P&O (Perturb and Observe), PID and T1FLC [34]. Verma et al. have developed an asymmetric interval Type-2 fuzzy logic controller for the MPPT algorithm to perform best performance, when photovoltaic (PV) array is partially irradiated under uniform solar irradiance. They compared their developed algorithm with other approaches, such as perturb & observe (P&O) and T1FLC, to assess for GMPP tracking, fill factor, shading losses, mismatch loss and efficiency [35].

4. Applications of T2FNs in solar energy

Type-2 fuzzy systems contribute to decision-making processes in solar energy projects by considering uncertainties and multiple criteria. This facilitates the formulation of effective and informed strategies in the planning and execution of solar initiatives. The FL approach is often integrated with many multi-criteria decision-making (MCDM) methods, offering application-centric approaches to complex decision-making challenges involving multiple criteria and inherent uncertainties. FL integrated standard techniques are referred to as fuzzy hybrid techniques. This integration approach is widely embraced to obtain comprehensive and realistic results when dealing with complex real-world decision-making problems. This is attributed to FL's robust utility as a powerful tool for effectively managing such uncertainties in numerous decision-making processes where the natural presence of uncertainty and imprecise information is acknowledged [19].

The theoretical foundations of MCDM processes, such as Technique for Analytical Network Process (ANP), Order Preference by Similarity to Ideal Solution (TOPSIS), and VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), Analytical Hierarchy Process (AHP), along with the arithmetic operations employed in these processes, are conducive to the utilization of FNs. Decision-making methods based on T1FNs are extensively applied in a diverse range of academic studies [15], [36]–[38], due to both the ease of calculations compared to other fuzzy numbers and their proficiency in addressing problems characterized by uncertainty [39] .

Methods based on T2FSs in MCDM mostly aim to make decisions considering the advantages and disadvantages of renewable energy alternatives (REAs) based on various selection criteria that encompass both quantitative and qualitative factors [39], [40]. In their study to determine the best RES for Turkey, Balin & Baraçli used the Interval Type-2 Fuzzy TOPSIS (IT2FTOPSIS) method to compare solar, biomass, geothermal, hydraulic and hydrogen renewable energy sources. According to the results of the study, the best alternative for energy investments in Turkey is wind energy, followed by solar, biomass, geothermal, hydraulic and hydrogen energies, respectively [41]. In a comparable study, Çolak et al. applied an integrated

MCDM model for prioritizing REAs in Turkey. In their suggested fuzzy MCDM model, they combine the Interval Type-2 Fuzzy Analytic Hierarchy Process (IT2AHP) method for determining the weights of decision criteria with the hesitant fuzzy TOPSIS method for prioritizing REAs. The study also includes a sensitivity analysis examine the effects of main criteria weights in ranking. When applied to Turkey, their proposed model ranks the RES as Hydraulic Energy, Wind Energy, Geothermal Energy, Solar Energy, Biomass Energy, Wave Energy, and Hydrogen Energy [42]. The specific characteristics of countries will also influence the comparative results of RESs, another study to compare RESs is the application of a Gaussian IT2FSs-based prospect theory method by He et al. The authors introduce a Gaussian interval type-2 fuzzy distance measure and A Gaussian interval type-2 fuzzy entropy model in their research. With their developed evaluation approach, they rank hydraulic, solar, biomass, wind and electrochemical energy types in Anhui, China. The study compares the results with the rankings of extended fuzzy TOPSIS-based RES evaluation method [43], interval type-2 fuzzy prospect theory method and IT2FTOPSIS [44].

The increasing sensitivity towards sustainability has become a factor influencing the choice of RES and alternative fuels [45]. Hendiani & Grit Walther propose a new MCDM method based on the concept of Interval Type-2 Fuzzy Ideal Solution Distance to assess the sustainability performance of renewable energy systems and determine the existing degree of sustainability. The notion of distance to the ideal solution is introduced and generalized with Interval Type-2 fuzzy sets. In the study, social, economic, and environmental aspects are identified as three separate indicators of sustainability. The results reveal that factors such as “Filling station availability” from an economic perspective, “NOx emissions”, “Need for waste disposal” and “Land requirements” from an environmental perspective and “Social acceptability” from a social perspective are identified as low-performing factors contributing significantly to both individual and overall sustainability performance [46]. In another study on sustainability, Abdullah & Najib applied an Interval Type-2 Fuzzy Analytic Hierarchy Process (IT2FAHP) through a seven-step calculation process to select a sustainable energy source among seven identified alternatives. The results indicate that solar energy emerges as the most viable alternative among sustainable energy sources [47].

Decision-making methods can be employed not only for the selection of RES, but also, as in the study by Li et al., to explore consumers’ expectations from RESs. Li et al. employed the Interval Type-2 Fuzzy DEMATEL (IT2FDEMATEL) method to determine the weights of strategies for solar energy investments based on the priorities assigned by customers to 8 different TRIZ-based innovative strategies. Through this approach, they identified the more significant strategies by weighting them. According to the results, the best TRIZ-based investment strategy for solar energy projects, for both commercial and non-commercial customers, emerged as replacing the mechanical system. [48]

Technical decisions can also be made using MDCM methods, such as the solar panels selection or the decision on a sun-tracking system in a solar energy facility. Tüysüz & Kahraman aimed to create a reliable three-dimensional decision environment by integrating Z-numbers into Picture Fuzzy Sets (PFS) in the literature to evaluate solar panel alternatives in Turkey. The AHP method was expanded with picture fuzzy Z-numbers to weigh evaluation criteria and the TOPSIS method was extended with picture fuzzy Z-numbers to prioritize the considered alternatives. The study revealed that Monocrystalline PERC is among the good solar panels, while Cadmium Telluride thin film is identified as the least favorable[49]. Umer et al. have introduced an expanded TOPSIS method in the context of Interval Type-2 Trapezoidal Pythagorean Fuzzy Numbers (IT2TrPFN). They compared widely used tracking systems, including Active Tracking (AT), Manual Tracking (MT) and Passive Tracking (PT), based on reliability, response and accuracy criteria [50].

5. Conclusion

In conclusion this book chapter collectively embark on a comprehensive exploration of T2FL methodologies specifically within the domain of solar energy research and delves into the intricate landscape of Type-2 Fuzzy Logic (T2FL) within the realm of solar energy research drawing inspiration from existing scholarly works. Leveraging insights from contemporary scientific literature the content presented in this volume contributes to a nuanced understanding of T2FL methodologies and their implications within the realm of solar technologies. The elucidation of numerical operations notably arithmetic intricacies and ranking dynamics within IT2FNs contributes to the foundational understanding of T2FL methodologies. By situating T2FL principles within the dynamic context of solar energy these chapters provide valuable perspectives for both academic and professional audiences. In summary this book chapter aspire to provide a holistic view of T2FL methodologies contextualizing their significance within the specific domain of solar energy research.

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