Chapter 3

λ-statistical Convergence in Intuitionistic Fuzzy Metric Spaces

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Abstract

Statistical convergence, built on the density of natural numbers, was independently defined by Steinhaus and Fast in 1951, has become one of the most active mathematics research areas. Many authors have recently thought about it in various metric spaces, and many valuable results have been obtained. In this work, we propose the notions of $\lambda-$statistical convergence and $\lambda-$statistical Cauchy sequences in intuitionistic fuzzy metric spaces. Afterward, we establish the relation between these concepts.

1.Introduction and Background

Convergence is one of the important concepts in mathematics. Many scientists have done and are doing many studies on this concept. Statistical convergence, which is a generalization of convergence in the ordinary sense, was first defined independently in 1951 by Fast [1] and Steinhaus [2]. Many scientists have worked on the concept of statistical convergence [3–8].

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Definition 1.1. [3] Let $(\mathbb{R}, |.|)$ be a metric space, (y_n) be a sequence in $\mathbb{R}$, and $y_0 \in \mathbb{R}$. Then, a sequence (y_n) is called statistically convergent to y_0, if, for all $\varepsilon > 0$,

$$\delta(\{n \in \mathbb{N} : |y_n - y_0| \geq \varepsilon\}) = 0.$$

The concept of $\lambda-$statistical convergence concept was proposed by Mursaalen [8] in 2000 as follows:

Definition 1.2. [8] Let $\lambda = (\lambda_k)$ be a non-decreasing sequence of positive numbers tending to ∞ such that $\lambda_{k+1} \leq \lambda_k + 1$, $\lambda_1 = 0$. Let $A \subseteq \mathbb{N}$. The number

$$\delta_\lambda(A) := \lim_k \frac{|\{t \in I_k : t \in A\}|}{\lambda_k}$$

is referred to be the λ-density of A, where $I_k = [k - \lambda_k + 1, k]$. If $\lambda_k = k$ for all k, then λ-density is reduced to the asymptotic density.

Definition 1.3. [8] Let $(\mathbb{R}, |.|)$ be a metric space, (y_n) be a sequence in $\mathbb{R}$, and $y_0 \in \mathbb{R}$. Then, a sequence (y_n) is referred to be $\lambda-$statistical convergent to the number y_0, if, for all $\varepsilon > 0$,

$$\delta_\lambda(\{n \in I_k : |y_n - y_0| \geq \varepsilon\}) = 0.$$

In this case we write $S_\lambda - \lim_{n\to\infty} y_n = y_0$.

Many people have worked to explain some conditions that mathematics could not explain in life. Fuzzy sets, which is one of them, was explained by Zadeh [9] in 1965. Here, who introduced the concepts of fuzzy set and degree of membership for the first time Zadeh, inclusion, intersection, union, complement operations of set theory with like correlation and convexity defined many properties.

Definition 1.4. [9] A fuzzy set (FS) F, defined on a universe of discourse E, is characterized by a membership function $\varphi_F(x)$ that assigns any element $x \in E$ a real valued grade of membership in F. By definition, the values $\varphi_F(x)$ may lie within the closed interval [0,1]. The FS is represented as: $F = \{(x, \varphi_F(x) : x \in E\}$.

In 1986, Atanassov [14] extended fuzzy set. By adding the idea of not belonging to the degree of belonging to the fuzzy set, he defined the intuitive fuzzy set concept, which is a generalization of the fuzzy set concept.

Definition 1.5. [14] An intuitionistic fuzzy sets (IFS) F, defined on a universe of discourse E, is characterized by a membership function $\varphi_F(x)$ and a non-membership function $\vartheta_F(x)$ for any element $x \in E$. The IFS is represented as: $F = \{(x, \varphi_F(x), \vartheta_F(x)) : x \in E\}$ where $\varphi_F(x) + \vartheta_F(x) \leq 1$.

Remark 1.6. [14] Every fuzzy set is obviously an intuitionistic fuzzy set.

Now, we recall some basic definitions such as t-norm, t-conorm and others besides some related properties given by Schweizer and Sklar [10].

Definition 1.7. [10] Let $T : [0,1]^2 \to [0,1]$ be a function. Then T is referred to be triangular norm (t-norm), if these axioms are satisfied: for all $k, l, m, n \in [0,1]$,

1. $T(k, 1) = k$
2. $T(k, l) = T(l, k)$
3. If $k \leq m, l \leq n$, then $T(k, l) \leq T(m, n)$,
4. $T(T(k, l), m) = T\big(k, T(l, m)\big)$.

Definition 1.8. [10] Let $S : [0,1]^2 \to [0,1]$ be a function. Then S is called triangular conorm (t-conorm), if these axioms are satisfied: for all $k, l, m, n \in [0,1]$,

1. $S(k, 0) = k$
2. $S(k, l) = S(l, k)$
3. If $k \leq m, l \leq n$, then $S(k, l) \leq S(m, n)$,
4. $S(S(k, l), m) = S\big(k, S(l, m)\big)$.

Example 1.9. [10] According to the previous two definitions, these operators are basic examples of t-norm and t-conorms, respectively.

1. $T(k, l) = kl$
2. $T(k, l) = \min \{k, l\}$
3. $S(k, l) = \max \{k, l\}$
4. $S(k, l) = \min \{k + l, 1\}$

In addition, fuzzy metric spaces (FMSs) extend metric spaces by introducing degrees of membership or fuzziness of points. Kramosil and

Michalek [11] and Kaleva and Seikkala [12] were among the first to investigate FMSs. Building on Kramosil and Michalek's [11] work, George and Veermani [13] redefined the concept of FMSs by utilizing a continuous t-norm and obtained the Haussdorf topology of these spaces. Lately, with the help of Definition 1.7 and 1.8; Park [15] has recently proposed intuitionistic fuzzy metric spaces (IFMS) as follows:

Definition 1.10. [15] Let $\mathbb{B}$ be an arbitrary set, T be a continuous t-norm, S be a continuous t-conorm, and φ, ϑ be fuzzy sets on $\mathbb{B}^2 \times (0, \infty)$. If φ and ϑ satisfy these conditions: for all $k, l, m \in \mathbb{B}$ and $u, s > 0$,

1. $\varphi(k, l, u) + \vartheta(k, l, u) \leq 1$
2. $\varphi(k, l, u) > 0$
3. $\varphi(k, l, u) = 1 \Leftrightarrow k = l$
4. $\varphi(k, l, u) = \varphi(l, k, u)$
5. $\varphi(k, m, u + s) \geq T(\varphi(k, l, u), \varphi(l, m, s))$
6. The function $\varphi(k, l, .) : (0, \infty) \to (0,1]$ is continuous
7. $\vartheta(k, l, u) > 0$
8. $\vartheta(k, l, u) = 0 \Leftrightarrow k = l$
9. $\vartheta(k, l, u) = \vartheta(l, k, u)$
10. $\vartheta(k, m, u + s) \leq S(\vartheta(k, l, u), \vartheta(l, m, s))$
11. The function $\vartheta(k, l, .) : (0, \infty) \to (0,1]$ is continuous

then a 5-tuple $(\mathbb{B}, \varphi, \vartheta, T, S)$ is called an intuitionistic fuzzy metric space.

The values $\varphi(k, l, u)$ and $\vartheta(k, l, u)$ stand for the degree of membership and non-membership between k and l concerning u, respectively.

Example 1.11. [15] Suppose $(\mathbb{B}, d)$ is a metric space. Define $T(k, l) = kl$ and $S(k, l)$ min $\{k + l, 1\}$ for all $k, l \in [0,1]$, and suppose that φ and ϑ are fuzzy sets on $\mathbb{B}^2 \times (0, \infty)$ defined as

$$\varphi(k, l, u) = \frac{u}{u + d(k, l)} \text{ and } \vartheta(k, l, u) = \frac{d(k, l)}{u + d(k, l)}$$

for $k, l \in \mathbb{B}$ and $u > 0$. Then $(\mathbb{B}, \varphi, \vartheta, T, S)$ is an IFMS.

In addition, convergence and Cauchy sequence in IFMS are as follows:

Definition 1.12. [15] Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. Then, a sequence (y_n) in $\mathbb{B}$ is said to be convergent to $y_0 \in \mathbb{B}$ concerning IFM (φ,ϑ), if, for all $\varepsilon \in (0,1)$ and $u > 0$, there exists $n_\varepsilon \in \mathbb{N}$ such that $n \geq n_\varepsilon$ implies

$$\varphi(y_n, y_0, u) > 1-\varepsilon \text{ and } \vartheta(y_n, y_0, u) < \varepsilon$$

or equivalently

$$\lim_{n\to\infty} \varphi(y_n, y_0, u) = 1 \text{ and } \lim_{n\to\infty} \vartheta(y_n, y_0, u) = 0$$

and is denoted by $\frac{\varphi}{\vartheta} - \lim_{n\to\infty} y_n = y_0$ or $y_n \overset{\varphi}{\underset{\vartheta}{\to}} y_0$ as $n \to \infty$.

Definition 1.13. [15] Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. Then, a sequence (y_n) is referred to as a Cauchy sequence in $\mathbb{B}$ concerning IFM (φ,ϑ), if, for all $u > 0$ and $\varepsilon \in (0,1)$, there exists $n_\varepsilon \in \mathbb{N}$ such that $n, N \geq n_\varepsilon$implies

$$\varphi(y_n, y_N, u) > 1-\varepsilon \text{ and } \vartheta(y_n, y_N, u) < \varepsilon$$

or equivalently

$$\lim_{n,N\to\infty} \varphi(y_n, y_N, u) = 1 \text{ and } \lim_{n,N\to\infty} \vartheta(y_n, y_N, u) = 0.$$

Statistical convergence and Cauchy sequence in IFMS was expressed in 2022 by Varol [16] as follows:

Definition 1.14. [16] Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. Then, a sequence (y_n) in $\mathbb{B}$ is called statistically convergent to $y_0 \in \mathbb{B}$ concerning IFM (φ,ϑ), if, for all $\varepsilon \in (0,1)$ and $u > 0$,

$$\delta(\{n \in \mathbb{N} : \varphi(y_n, y_0, u) \leq 1-\varepsilon \text{ or } \vartheta(y_n, y_0, u) \geq \varepsilon\}) = 0$$

or equivalently

$$\lim_{n\to\infty} \frac{|\{k \in \mathbb{N} : (k \leq n) \text{ and } (\varphi(y_k, y_0, u) \leq 1-\varepsilon \text{ or } \vartheta(y_k, y_0, u) \geq \varepsilon)\}|}{n} = 0.$$

Definition 1.15. [16] Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. Then, a sequence (y_n) is referred to be a statistical Cauchy sequence in $\mathbb{B}$ concerning IFM (φ,ϑ), if, for all $\varepsilon \in (0,1)$ and $u > 0$, there exists $N \in \mathbb{N}$ such that

$$\delta(\{n \in \mathbb{N} : \varphi(y_n, y_N, u) \leq 1-\varepsilon \text{ or } \vartheta(y_n, y_N, u) \geq \varepsilon\}) = 0.$$

2.Main Results

This section defines $\lambda-$statistical convergence, $\lambda-$statistical Cauchy sequences in IFMSs. In addition, it provides some of basic properties.

Definition 2.1. Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. Then, a sequence (y_n) in $\mathbb{B}$ is said to be $\lambda-$statistically convergent to $y_0 \in \mathbb{B}$ concerning IFM (φ,ϑ), if, for all $\varepsilon \in (0,1)$ and $u > 0$,

$$\delta_\lambda(\{n \in I_k : \varphi(y_n, y_0, u) > 1-\varepsilon \text{ and } \vartheta(y_n, y_0, u) < \varepsilon\}) = 1$$

or equivalently

$$\delta_\lambda(\{n \in I_k : \varphi(y_n, y_0, u) \leq 1-\varepsilon \text{ or } \vartheta(y_n, y_0, u) \geq \varepsilon\}) = 0$$

and is denoted by ${}^{\varphi}_{\vartheta}S_\lambda - \lim_{n\to\infty} y_n = y_0$ or $y_n \overset{{}^{\varphi}_{\vartheta}S_\lambda}{\longrightarrow} y_0$ as $n \to \infty$.

Example 2.2. Let $\mathbb{B}=\mathbb{R}$, $T(m_1,m_2)=m_1m_2$, and $S(m_1,m_2)=\min\ \{m_1+m_2,1\}$ for all $m_1,m_2 \in [0,1]$. Define φ and ϑ by

$$\varphi(k,l,u) = \frac{u}{u+|k-l|} \text{ and } \vartheta(k,l,u) = \frac{|k-l|}{u+|k-l|}$$

for all $k,l \in \mathbb{R}$ and $u > 0$. Then, $(\mathbb{R},\varphi,\vartheta,T,S)$ is an IFMS. Now define a sequence (y_t) by

$$y_t = \begin{cases} t, & \text{if } k-\left[\sqrt{\lambda_k}\right]+1 \leq t \leq k \\ 0, & \text{otherwise} \end{cases}$$

Let

$$K(u,\varepsilon) = \{t \in I_k : \varphi(y_t,0,u) \leq 1-\varepsilon \text{ or } \vartheta(y_t,0,u) \geq \varepsilon\}$$

for $\varepsilon \in (0,1)$ and $u > 0$. Then

$$\begin{aligned} K(u,\varepsilon) &= \left\{t \in I_k : \frac{u}{u+|y_t|} \leq 1-\varepsilon \text{ or } \frac{|y_t|}{u+|y_t|} \geq \varepsilon\right\} \\ &= \left\{t \in I_k : |y_t| \geq \frac{\varepsilon u}{1-\varepsilon} > 0\right\} \\ &= \{t \in I_k : |y_t| = t\} \\ &= \left\{t \in I_k :\ k-\left[\sqrt{\lambda_k}\right]+1 \leq t \leq k\right\} \end{aligned}$$

and therefore, we get

$$\frac{|K(u,\varepsilon)|}{\lambda_k} = \frac{\left|\left\{t \in I_k :\ k-\left[\sqrt{\lambda_k}\right]+1 \leq t \leq k\right\}\right|}{\lambda_k} \leq \frac{\sqrt{\lambda_k}}{\lambda_k}$$

which implies that $\lim_{k\to\infty} \frac{|K(u,\varepsilon)|}{\lambda_k} = 0$. Hence, $\delta_\lambda\big(K(u,\varepsilon)\big) = 0$ implies that $y_t \xrightarrow{{}^{\varphi}_{\vartheta}S_\lambda} 0$ as $t \to \infty$.

Lemma 2.3. Let $(\mathbb{B}, \varphi, \vartheta, T, S)$ be an IFMS, (y_n) be a sequence in $\mathbb{B}$, and $y_0 \in \mathbb{B}$. Then, for all $\varepsilon \in (0,1)$ and $u > 0$, the following are equivalent:

1. ${}^{\varphi}_{\vartheta}S_\lambda - \lim_{n\to\infty} y_n = y_0$;
2. $\delta_\lambda(\{n \in I_k : \varphi(y_n, y_0, u) \leq 1 - \varepsilon\}) = \delta_\lambda(\{n \in I_k : \vartheta(y_n, y_0, u) \geq \varepsilon\}) = 0$;
3. $\delta_\lambda(\{n \in I_k : \varphi(y_n, y_0, u) > 1 - \varepsilon\}) = \delta_\lambda(\{n \in I_k : \vartheta(y_n, y_0, u) < \varepsilon\}) = 1$.

Proof. It can be straightforwardly proved using Definition 2.1 and the density function's properties.

Theorem 2.4. Let $(\mathbb{B}, \varphi, \vartheta, T, S)$ be an IFMS. If a sequence (y_n) in $\mathbb{B}$ is $\lambda-$statistically convergent concerning IFM (φ, ϑ), then its limit is unique.

Proof. Suppose that ${}^{\varphi}_{\vartheta}S_\lambda - \lim_{n\to\infty} y_n = y_1$, ${}^{\varphi}_{\vartheta}S_\lambda - \lim_{n\to\infty} y_n = y_2$, and $y_1 \neq y_2$. For a given $\varepsilon \in (0,1)$, choose $\kappa \in (0,1)$ such that $T(1-\kappa, 1-\kappa) > 1 - \varepsilon$ and $S(\kappa, \kappa) < \varepsilon$. Then, for all $u > 0$, let

$$K_1(\kappa, u) := \{n \in I_k : \varphi(y_n, y_1, u) \leq 1 - \kappa\},$$
$$K_2(\kappa, u) := \{n \in I_k : \varphi(y_n, y_2, u) \leq 1 - \kappa\},$$
$$L_1(\kappa, u) := \{n \in I_k : \vartheta(y_n, y_1, u) \geq \kappa\},$$
$$L_2(\kappa, u) := \{n \in I_k : \vartheta(y_n, y_2, u) \geq \kappa\}.$$

So from (y_n) is λ-statistically convergent to y_1 and Lemma (2.3),

$$\delta_\lambda\big(K_1(\kappa, u)\big) = 0 \text{ and } \delta_\lambda\big(L_1(\kappa, u)\big) = 0.$$

Also using (y_n) is λ-statistically convergent to y_2 and Lemma (2.3),

$$\delta_\lambda\big(K_2(\kappa, u)\big) = 0 \text{ and } \delta_\lambda\big(L_2(\kappa, u)\big) = 0.$$

Let

$$K_{\varphi\vartheta}(\kappa, u) := \big(K_1(\kappa, u) \cup K_2(\kappa, u)\big) \cap \big(L_1(\kappa, u) \cup L_2(\kappa, u)\big).$$

Hence, $\delta_\lambda\left(K_{\varphi\vartheta}(\kappa,u)\right)=0$ which implies that $\delta_\lambda\left(\mathbb{N}\setminus K_{\varphi\vartheta}(\kappa,u)\right)=1$. If $n\in\mathbb{N}\setminus K_{\varphi\vartheta}(\kappa,u)$, then we have two options:

$$n\in\mathbb{N}\setminus\left(K_1(\kappa,u)\cup K_2(\kappa,u)\right) \text{ or } n\in\mathbb{N}\setminus\left(L_1(\kappa,u)\cup L_2(\kappa,u)\right)$$

Let us consider $n\in\mathbb{N}\setminus\left(K_1(\kappa,u)\cup K_2(\kappa,u)\right)$. Then, we obtain

$$\varphi(y_1,y_2,u)\geq T\left(\varphi\left(y_1,y_n,\frac{u}{2}\right),\varphi\left(y_n,y_2,\frac{u}{2}\right)\right)>T(1-\kappa,1-\kappa)>1-\varepsilon$$

Therefore, $\varphi(y_1,y_2,u)>1-\varepsilon$ and since $\varepsilon\in(0,1)$ is arbitrary, $\varphi(y_1,y_2,u)=1$ for all $u>0$, which implies $y_1=y_2$.
Now, let us consider $n\in\mathbb{N}\setminus\left(L_1(\kappa,u)\cup L_2(\kappa,u)\right)$. Then,

$$\vartheta(y_1,y_2,u)\leq S\left(\vartheta\left(y_1,y_n,\frac{u}{2}\right),\vartheta\left(y_n,y_2,\frac{u}{2}\right)\right)<S(\kappa,\kappa)<\varepsilon$$

Since $\varepsilon\in(0,1)$ is arbitrary, we obtain $\vartheta(y_1,y_2,u)=1$ for all $u>0$, which implies $y_1=y_2$.

Theorem 2.5. Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS and (y_n) be a sequence in $\mathbb{B}$. If (y_n) is convergent to $y_0\in\mathbb{B}$ concerning IFM (φ,ϑ), then (y_n) is $\lambda-$statistically convergent to y_0 concerning IFM (φ,ϑ).

Proof. Let (y_n) be convergent to $y_0\in\mathbb{B}$. Then, for all $\varepsilon\in(0,1)$ and $u>0$, there exists $n_0\in\mathbb{N}$ such that $\varphi(y_n,y_0,u)>1-\varepsilon$ and $\vartheta(y_n,y_0,u)<\varepsilon$. Hence, the set

$$A(\varepsilon)=\{n\in\mathbb{N}:\ \varphi(y_n,y_0,u)\leq 1-\varepsilon \text{ or } \vartheta(y_n,y_0,u)\geq\varepsilon\}$$

has a finite number of terms. Also,

$$A(\varepsilon)\supset\{n\in I_k:\ \varphi(y_n,y_0,u)\leq 1-\varepsilon \text{ or } \vartheta(y_n,y_0,u)\geq\varepsilon\}.$$

Consequently,

$$\delta_\lambda(\{n\in I_k:\varphi(y_n,y_0,u)\leq 1-\varepsilon \text{ or } \vartheta(y_n,y_0,u)\geq\varepsilon\})=0$$

The converse of the theorem is not always hold.

Example 2.6. Let $\mathbb{B}=\mathbb{R}$, $T(m_1,m_2)=m_1m_2$, and $S(m_1,m_2)=\min\ \{m_1+m_2,1\}$ for all $m_1,m_2\in[0,1]$. Define φ and ϑ by

$$\varphi(k,l,u)=\frac{u}{u+|k-l|} \text{ and } \vartheta(k,l,u)=\frac{|k-l|}{u+|k-l|}$$

for all $k,l\in\mathbb{R}$ and $u>0$. Then, $(\mathbb{R},\varphi,\vartheta,T,S)$ is an IFMS. Now define a sequence (y_t) by

$$y_t=\begin{cases} t, & \text{if } k-\left[\sqrt{\lambda_k}\right]+1\le t\le k \\ 0, & \text{otherwise}\end{cases}$$

From Example 2.2 (y_t) is $\lambda-$statistically convergent to 0. On the other hand, (y_t) is not convergent to 0 with the respect to IFM (φ,ϑ), since

$$\varphi(y_t,0,u)=\frac{u}{u+|y_t|}=\begin{cases} \dfrac{u}{u+t}, & \text{if } k-\left[\sqrt{\lambda_k}\right]+1\le t\le k \\ 0, & \text{otherwise}\end{cases}\le 1$$

and

$$\vartheta(y_t,0,u)=\frac{|y_t|}{u+|y_t|}=\begin{cases} \dfrac{t}{u+t}, & \text{if } k-\left[\sqrt{\lambda_k}\right]+1\le t\le k \\ 0, & \text{otherwise}\end{cases}\ge 0.$$

Definition 2.7. Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. Then, a sequence (y_n) is referred to be a $\lambda-$statistically Cauchy sequence in $\mathbb{B}$ concerning IFM (φ,ϑ), if, for all $\varepsilon\in(0,1)$ and $u>0$, there exists $m\in\mathbb{N}$ such that

$$\delta_\lambda(\{n\in I_k : \varphi(y_n,y_m,u)>1-\varepsilon \text{ and } \vartheta(y_n,y_m,u)<\varepsilon\})=1$$

or equivalently

$$\delta_\lambda(\{n\in I_k : \varphi(y_n,y_m,u)\le 1-\varepsilon \text{ or } \vartheta(y_n,y_m,u)\ge\varepsilon\})=0.$$

Theorem 2.8. Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS and (y_n) be a sequence in $\mathbb{B}$. Then, a sequence (y_n) is a Cauchy sequence concerning IFM (φ,ϑ), then it is a $\lambda-$ statistical Cauchy sequence concerning IFM (φ,ϑ).

Proof. The proof is similar to Theorem 2.5.

Theorem 2.9. Let $(\mathbb{B},\varphi,\vartheta,T,S)$ be an IFMS. A sequence (y_n) is $\lambda-$statistically convergent concerning IFM (φ,ϑ), then it is a $\lambda-$statistically Cauchy concerning IFM (φ,ϑ).

Proof. Let (y_n) be $\lambda-$statistical convergent to y_0 concerning IFM (φ,ϑ), i.e. ${}^{\varphi}_{\vartheta}S_\lambda-\lim_{n\to\infty}y_n=y_0$. For given $r\in(0,1)$, choose $\varepsilon\in$

$(0,1)$ such that $T(1-r,1-r)>1-\varepsilon$ and $S(r,r)<\varepsilon$. Then, for $u>0$, we have

$$\delta_\lambda(A(r,u)) = \delta_\lambda\left(\left\{n\in I_k : \varphi\left(y_n,y_0,\frac{u}{2}\right)<1-r \text{ and } \vartheta\left(y_n,y_0,\frac{u}{2}\right)<r\right\}\right) = 1.$$

Let $m\in A(r,u)$. Then $\varphi\left(y_m,y_0,\frac{u}{2}\right)>1-r$ and $\vartheta\left(y_m,y_0,\frac{u}{2}\right)>r$. Hence,

$$\varphi(y_n,y_m,u)\geq T\left(\varphi\left(y_n,y_0,\frac{u}{2}\right),\varphi\left(y_0,y_m,\frac{u}{2}\right)\right)>T(1-r,1-r)>1-\varepsilon$$

and

$$\vartheta(y_n,y_m,u)\leq S\left(\vartheta\left(y_n,y_0,\frac{u}{2}\right),\vartheta\left(y_0,y_m,\frac{u}{2}\right)\right)<S(r,r)<\varepsilon.$$

Therefore,

$$m\in B(r,u)=\{n\in I_k : \varphi(y_n,y_m,u)>1-r \text{ and } \vartheta(y_n,y_m,u)<r\}.$$

Consequently, (y_n) is a $\lambda-$statistically Cauchy sequence concerning IFM (φ,ϑ).

3.Conclusion

This paper deals with the concept of lambda statistical convergence in intuitionistic fuzzy metric space. In addition, it investigates the concept of lambda statistical Cauchy sequence and the basic properties of this concept.

In further work, we also believe that lambda convergence for double sequences can be defined using the concepts in intuitionistic fuzzy metric space and results presented here, and its basic properties can be studied.

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